Calculators are not allowed

Answer the following questions:

1. (a) (2+2 points) Evaluate each of the following limits, if it exists

     (i) $\lim\limits_{x\to 0} \dfrac{|\sin x|}{x}$,   (ii) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{x^2-1}}{x}$

    (b) (4 points) Find the derivative of

$$y = \sec^3\left(\frac{1+x}{\sqrt{x}}\right).$$

2. (a) (4 points) Find the extrema of the function

$$f(x) = (x-2)^{\frac{2}{3}} - 5$$

     in the interval $[-6, 3]$.

    (b) (4 points) Use the mean value theorem to show that if $f'(x) > 0$ on the interval (a, b), then f is increasing on (a, b).

3. Let

$$f(x) = \frac{x-2}{x+5}.$$

    (a) (2 points) Find the intervals on which f is increasing or is decreasing, and find the local extrema of f (if any).

    (b) (2 points) Find the intervals on which the graph of f is concave upward or concave downward, and find the points of inflection (if any).

    (c) (2 points) Find the vertical and horizontal asymptotes for the graph of f (if any).

    (d) (2 points) Sketch the graph of f.

4. (4+2+4 points) Evaluate the following integrals,

    (a) $\displaystyle\int \frac{\sqrt{x^6 + 5x^4}}{x}\, dx$, $x > 0$.   (b) $\displaystyle\int \pi^2\, dx$.   (c) $\displaystyle\int_0^6 |x - 3|\, dx$.

5. (a) (5 points) Find the area of the region bounded by the graphs of $y = x^2$ and $y = 8 - x^2$.

    (b) (5 points) Let

$$f(x) = \int_1^x \sqrt{t^2 - 1}\, dt,$$

    find the arc length of the graph of the equation $y = f(x)$ from $A(1, f(1))$ to $B(2, f(2))$.

6. (6 points) The region in the first quadrant bounded by the graphs of the curves $y = \dfrac{2}{x}$ and $x + y = 3$ is revolved about the x-axis. Find the volume of the resulting solid.

(Good Luck)